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Magnon Bose–Einstein condensation and spin superfluidity

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Abstract
Bose–Einstein condensation (BEC) is a quantum phenomenon of formation of a collective quantum state in which a macroscopic number of particles occupy the lowest energy state and thus is governed by a single wavefunction. Here we highlight the BEC in a magnetic subsystem—the BEC of magnons, elementary magnetic excitations. The magnon BEC is manifested as the spontaneously emerging state of the precessing spins, in which all spins precess with the same frequency and phase even in an inhomogeneous magnetic field. The coherent spin precession was observed first in superfluid ^{3}He-B and this domain was called the homogeneously precessing domain (HPD). The main feature of the HPD is the induction decay signal, which ranges over many orders of magnitude longer than is prescribed by the inhomogeneity of magnetic field. This means that spins precess *not* with a local Larmor frequency, but coherently with a common frequency and phase. This BEC can also be created and stabilized by continuous NMR pumping. In this case the NMR frequency plays the role of a magnon chemical potential, which determines the density of the magnon condensate. The interference between two condensates has also been demonstrated. It was shown that HPD exhibits all the properties of spin superfluidity. The main property is the existence of a spin supercurrent. This spin supercurrent flows separately from the mass current. Transfer of magnetization by the spin supercurrent by a distance of more than 1 cm has been observed. Also related phenomena have been observed: the spin current Josephson effect; the phase-slip processes at the critical current; and the spin current vortex—a topological defect which is the analog of a quantized vortex in superfluids and of an Abrikosov vortex in superconductors; and so on. It is important to mention that the spin supercurrent is a magnetic phenomenon, which is not directly related to the mass superfluidity of ^{3}He: it is the consequence of a specific antiferromagnetic ordering in superfluid ^{3}He. Several different states of coherent precession have been observed in ^{3}He-B: the homogeneously precessing domain (HPD); a persistent signal formed by Q-balls at very low temperatures; coherent precession with fractional magnetization; and two new modes of coherent precession in compressed aerogel. In compressed aerogel the coherent precession has been also found in ^{3}He-A. We demonstrate that the coherent precession of magnetization is a true BEC of magnons, with the magnon interaction term in the Gross–Pitaevskii equation being provided by spin–orbit coupling which is different for different states of the magnon BEC.

1. Introduction

The last decade has been marked by fundamental studies of mesoscopic quantum states of dilute ultracold atomic gases in the regime where the de Broglie wavelength of the atoms is comparable with their spacing, giving rise to the phenomenon of Bose–Einstein condensation (see, e.g., [1]). The formation of the Bose–Einstein condensate (BEC)—an accumulation of a macroscopic number of particles in the lowest energy state—was predicted by Einstein in 1925. In an ideal gas, all atoms are in the lowest energy state in the limit of zero temperature. In dilute atomic gases, weak interactions between atoms produces a small fraction of non-condensed atoms.

In the only known bosonic liquid, ^{4}He, which remains liquid at zero temperature, the BEC is strongly modified by interactions. The depletion of the condensate due to interactions is very strong: in the limit of zero temperature only about 10% of the particles occupy the state with zero momentum. Nevertheless, BEC still remains the key mechanism for the phenomenon of superfluidity in liquid ^{4}He: due to BEC the whole liquid (100% of the ^{4}He atoms) forms a coherent quantum state at $T = 0$ and participates in the non-dissipative superfluid flow.

Superfluidity is a very general quantum property of matter at low temperatures, with a variety of mechanisms and possible non-dissipative superfluid currents. These include: supercurrent of the electric charge in superconductors and mass supercurrent in superfluid ^{3}He, where the mechanism of superfluidity is the Cooper pairing; hypercharge supercurrent in the vacuum of the standard model of elementary particle physics, which comes from the Higgs mechanism; supercurrent of color charge in dense quark matter in quantum chromodynamics; etc. All of these supercurrents have the same origin: the spontaneous breaking of the $U(1)$ symmetry related to the conservation of the corresponding charge or particle number, which leads to the so called off-diagonal long-range order.

Formally, the phenomenon of BEC and superfluidity requires the conservation of charge or particle number. However, the consideration can be extended to systems with a weakly violated conservation law, including a system of sufficiently long-lived quasiparticles—discrete quanta of energy that can be treated as real particles in condensed matter. Here we shall consider the spin superfluidity—superfluidity in a magnetic subsystem of condensed matter—which is represented by BEC of magnons (quanta of excitations of the magnetic subsystem), and is manifested as the spontaneous phase coherent precession of spins first discovered in 1984 [2, 3].

2. BEC of quasiparticles

At high temperatures, spins of atoms are in a disordered paramagnetic state, which is similar to the high temperature phase of a weakly interacting gas. With cooling, the magnetic subsystem typically experiences a transition into an ordered state, in which magnetic moments are correlated at long distances. In cases when the magnetic $U(1)$ symmetry is spontaneously broken, some people describe this phenomenon in terms of BEC of magnons [4–6]. Let us stress from the beginning that there is the principal difference between the magnetic ordering and the BEC of quasiparticles which we are discussing in this review:

(i) In some magnetic systems, the symmetry breaking phase transition starts when the system becomes softly unstable towards growth of one of the magnon modes. The condensation of this mode leads finally to the formation of the true equilibrium ordered state. In the same manner, the Bose condensation of phonon modes may serve as a soft mechanism of formation of the equilibrium solid crystals [7]. But this does not mean that the final crystal state is the Bose condensate of phonons. On the contrary, BEC of quasiparticles is in principle a non-equilibrium phenomenon, since quasiparticles (magnons) have a finite lifetime. In our case magnons live long enough to form a state very close to the thermodynamic equilibrium BEC, but still it is not an equilibrium. In the final equilibrium state at $T = 0$ all the magnons will die out.

(ii) The ordered magnetic states are static equilibrium states which have diagonal long-range order. The magnon BEC is a dynamic state characterized by the off-diagonal long-range order, which is the main signature of spin superfluidity.

2.1. *Non-conservation versus coherence*

To prove that BEC of quasiparticles does really occur in a magnetic system, one should demonstrate the spontaneous emergence of coherence, and show the consequence of the coherence: spin superfluidity, which in particular includes the observation of interference between two condensates.

We shall demonstrate that the finite lifetime of magnons and the non-conservation of spin due to the spin–orbital coupling do not prevent the coherence of the magnon BEC. The gas of magnons can live a relatively long time, particularly at very low temperatures, sufficient for formation of a coherent magnon condensate. The non-conservation leads to a decrease of the number of magnons in the Bose gas until it disappears completely, but during this relaxation the coherence of BEC is preserved with all the signatures of spin superfluidity: (i) spin supercurrent, which transports the magnetization over a macroscopic distance more than 1 cm long; (ii) the spin current Josephson effect which shows interference between two condensates; (iii) phase-slip processes at the critical current; (iv) the spin current vortex—a topological defect which is an analog of a quantized vortex in superfluids, of an Abrikosov vortex in superconductors, and of cosmic strings in relativistic theories; (v) Goldstone modes related to the broken $U(1)$ symmetry—phonons in the spin superfluid magnon gas; etc.

The losses of spin and energy in the magnon BEC can be compensated by pumping of additional magnons. In this way one obtains a steady non-equilibrium state, which in our case is very close to the BEC in the thermodynamic equilibrium because the losses are relatively small.

3. Coherent Larmor precession as magnon BEC

3.1. *Disordered and coherent states of spin precession*

The magnetic subsystem which we discuss is the precessing magnetization. In full correspondence with atomic systems, the precessing spins can be either in the normal state or in the ordered spin superfluid state. In the normal state, spins

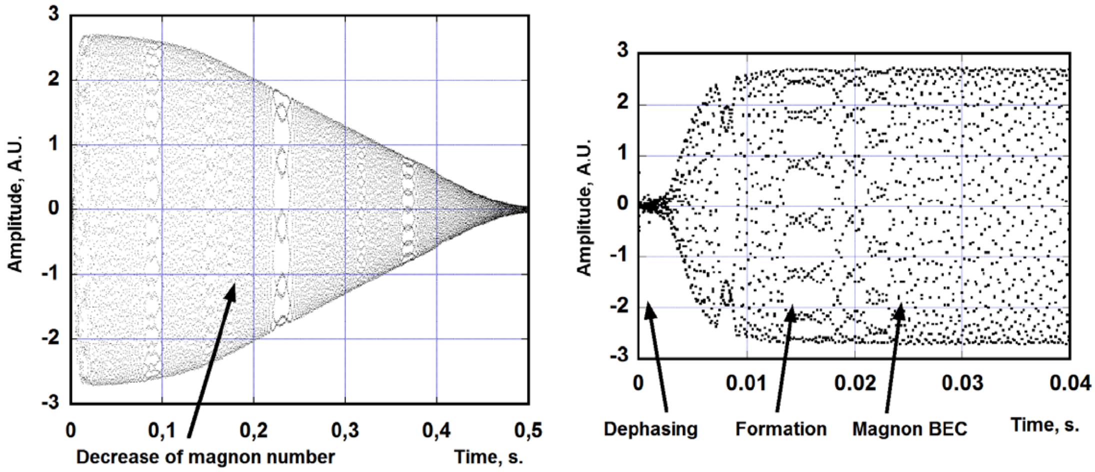

Figure 1. The stroboscopic record of the induction decay signal for a frequency about 1 MHz. Left: during the first stage of about 0.002 s the induction signal completely disappears due to dephasing. Then, during about 0.02 s, the spin supercurrent redistributes the magnetization and creates the phase coherent precession, which is equivalent to the magnon BEC state. Due to the small magnetic relaxation, the number of magnons slowly decreases but the precession remains coherent. Right: the initial part of the magnon BEC signal.

of atoms are precessing with the local frequency determined by the local magnetic field and interactions. In the ordered state the precession of all spins is coherent: they spontaneously develop the common global frequency and the global phase of precession.

In NMR experiments the magnetization is created by an applied static magnetic field: $\mathbf{M} = \chi\mathbf{H}$, where χ is the magnetic susceptibility. Then a pulse of the radio-frequency (RF) field $\mathbf{H}_{\mathrm{RF}} \perp \mathbf{H}$ deflects the magnetization by an angle β, and after that the induction signal from the free precession is measured. In the state of the disordered precession, spins almost immediately lose the information on the original common phase and frequency induced by the RF field, and due to this decoherence the measured induction signal is very short. In the ordered state, all spins precess coherently, which means that the whole macroscopic magnetization of the sample of volume V is precessing:

$$\mathcal{M}_x + \mathrm{i}\mathcal{M}_y = \mathcal{M}_\perp \mathrm{e}^{\mathrm{i}\omega t+\mathrm{i}\alpha}, \qquad \mathcal{M}_\perp = \chi H V \sin\beta. \quad (1)$$

This coherent precession is manifested as a huge and long-lived induction signal. In figure 1 we show the stroboscopic record of induction signal. After a fast decoherence due to inhomogeneity of magnetic field the spins spontaneously form a new state with coherent precession. Figure 2 shows the instant spectrum of the signal. In a short time after the RF pulse the spectrum of the signal corresponds to the inhomogeneity of the field in the experiment, 600 Hz. After the BEC formation the spectrum of the signal shrinks to only about 0.5 Hz. This shrinking by 1000 times of the spectral line corresponds to the almost 100% condensation of magnons into the coherent BEC state. The rest width of the induction signal is due to small magnetic relaxation. With time, the disorder increases, leading to increasing of the linewidth.

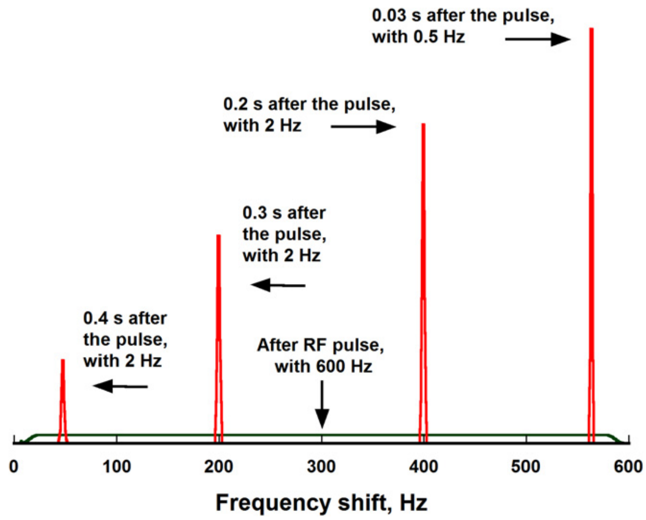

Figure 2. The instantaneous spectrum of the HPD induction signal immediately after the magnon excitation and after 0.03, 0.2, 0.3 and 0.4 s.

(This figure is in colour only in the electronic version)

3.2. Off-diagonal long-range order

The superfluid atomic systems are characterized by the off-diagonal long-range order (ODLRO) [8]. In superfluid ^{4}He and in the coherent atomic systems the operator of annihilation of atoms with momentum $\mathbf{p} = 0$ has a non-zero vacuum expectation value:

$$\langle \hat{a}_0 \rangle = \mathcal{N}_0^{1/2} \mathrm{e}^{\mathrm{i}\mu t+\mathrm{i}\alpha}, \quad (2)$$

where $\mathcal{N}_0$ is the number of particles in the Bose condensate, which in the limit of weak interactions between the atoms coincides at $T = 0$ with the total number of atoms $\mathcal{N}$. Equation (1) demonstrates that in the coherent precession, the ODLRO is manifested by a non-zero vacuum expectation value of the operator of creation of spin:

$$\langle \hat{S}_+ \rangle = \mathcal{S}_x + \mathrm{i}\mathcal{S}_y = \frac{\mathcal{M}_\perp}{\gamma} \mathrm{e}^{\mathrm{i}\omega t+\mathrm{i}\alpha}, \quad (3)$$

where γ is the gyromagnetic ratio. In the coherent spin precession the role of the particle number $\mathcal{N}$ is played by the projection $\mathcal{S}_z$ of the total spin on the direction of the magnetic field. The symmetry group $U(1)$ in magnetic systems is the group $O(2)$ of spin rotations about the direction of the magnetic field. This quantity $\mathcal{S}_z$ is conserved in the limit of vanishing spin–orbit coupling.

The spin–orbit interactions transform the spin angular momentum of the magnetic subsystem to orbital angular momentum, which causes losses of the spin $\mathcal{S}_z$ during the precession. In our system of superfluid ^{3}He, the spin–orbit coupling is relatively rather small, and thus $\mathcal{S}_z$ is quasi-conserved. Because of the losses of spin the precession will finally decay, but during its long lifetime the precession remains coherent; see figure 1. This is similar to the non-conservation of the number of atoms in the laser traps, where the number of atoms decreases with time but this does not destroy the coherence of the atomic BEC.

The ODLRO in (3) can be represented in terms of magnon condensation. In the second-quantization language magnons can be treated as weakly interacting particles obeying Bose–Einstein statistics. The operators of creation and annihilation of magnons are obtained from the spin operators using the Holstein–Primakoff transformation:

$$\hat{a}_0\sqrt{1-\frac{\hbar a_0^\dagger a_0}{2\mathcal{S}}}=\frac{\hat{\mathcal{S}}_+}{\sqrt{2\mathcal{S}\hbar}}, \qquad (4)$$

$$\hat{a}_0^\dagger\sqrt{1-\frac{\hbar a_0^\dagger a_0}{2\mathcal{S}}}=\frac{\hat{\mathcal{S}}_-}{\sqrt{2\mathcal{S}\hbar}}, \qquad (5)$$

$$\hat{\mathcal{N}}=\hat{a}_0^\dagger\hat{a}_0=\frac{\mathcal{S}-\hat{\mathcal{S}}_z}{\hbar}. \qquad (6)$$

Equation (6) relates the number of magnons $\mathcal{N}$ to the deviation of the spin $\mathcal{S}_z$ from its equilibrium value $\mathcal{S}_z^{(\mathrm{equilibrium})}=\mathcal{S}=\chi HV/\gamma$. In the full thermodynamic equilibrium, magnons are absent. Each magnon has spin $-\hbar$, and thus the total spin projection after pumping of $\mathcal{N}$ magnons into the system by the RF pulse is reduced by the number of magnons, $\mathcal{S}_z=\mathcal{S}-\hbar\mathcal{N}$. The ODLRO in magnon BEC is given by equation (2), where $\mathcal{N}_0=\mathcal{N}$ is the total number of magnons (6) in the BEC:

$$\langle\hat{a}_0\rangle=\mathcal{N}^{1/2}\mathrm{e}^{\mathrm{i}\omega t+\mathrm{i}\alpha}=\sqrt{\frac{2\mathcal{S}}{\hbar}}\sin\frac{\beta}{2}\mathrm{e}^{\mathrm{i}\omega t+\mathrm{i}\alpha}. \qquad (7)$$

Comparing (7) and (2), one finds that the role of the chemical potential in atomic systems μ is played by the global frequency of the coherent precession ω, i.e. $\mu\equiv\omega$. This demonstrates that the analogy with the phenomenon of BEC in atomic gases holds only for the dynamic states of a magnetic subsystem—states of precession.

This is the main difference from the reorientation transitions in magnetically ordered systems, which are sometimes treated as a Bose–Einstein condensation. Although the ordered magnetic systems discussed in [4–6] belong to the same class of spontaneously broken $U(1)$ symmetry as the magnon Bose condensate, these states are fully in equilibrium, and thus the chemical potential of magnons is strictly zero, $\mu=0$. Therefore such ordered states do not represent the true Bose condensation of magnons, as we discussed in section 2. Moreover, the magnetic order in these systems is diagonal, as distinct from the ODLRO in magnon BEC.

There are two approaches to studying the thermodynamics of atomic systems: one fixes either the particle number N or the chemical potential μ. For the magnon BEC, these two approaches correspond to two different experimental situations: pulsed and continuous NMR, respectively. In the case of free precession after the pulse, the number of magnons pumped into the system is conserved (if one neglects the losses of spin). This corresponds to the situation with the fixed $\mathcal{N}$, in which the system itself will choose the global frequency of the coherent precession (the magnon chemical potential). The opposite case is that of continuous NMR, when a small RF field is continuously applied to compensate the losses. In this steady state the frequency of precession is fixed by the frequency of the RF field, $\mu\equiv\omega=\omega_{\mathrm{RF}}$, and now the number of magnons will be adjusted to this frequency to match the resonance condition. In the precessing frame the steady state of coherent precession is static, if one neglects the spin–orbit interaction which couples orbital and spin frames.

3.3. The order parameter and the Gross–Pitaevskii equation

As in the case of the atomic Bose condensates the main physics of the magnon BEC can be found from the consideration of the Gross–Pitaevskii equation. The local complex order parameter is obtained by extension of equation (7) to the inhomogeneous case:

$$\Psi(\mathbf{r},t)=\langle\hat{\Psi}(\mathbf{r},t)\rangle, \qquad n=|\Psi|^2, \qquad \mathcal{N}=\int \mathrm{d}^3r|\Psi|^2, \qquad (8)$$

where n is magnon density. The Gross–Pitaevskii equation has the conventional form ($\hbar=1$):

$$-\mathrm{i}\frac{\partial\Psi}{\partial t}=\frac{\delta\mathcal{F}}{\delta\Psi^*}, \qquad (9)$$

where $\mathcal{F}\{\Psi\}$ is the free energy functional. In the coherent precession, the global frequency is constant in space and time (if dissipation is neglected): $\Psi(\mathbf{r},t)=\Psi(\mathbf{r})\mathrm{e}^{\mathrm{i}\omega t}$, and the Gross–Pitaevskii equation transforms into the Ginzburg–Landau equation: $\delta\mathcal{F}/\delta\Psi^*-\mu\Psi=0$ with $\mu=\omega$. The Ginzburg–Landau free energy functional has the following general form:

$$\mathcal{F}-\mu\mathcal{N}=\int \mathrm{d}^3r \times\left(\frac{|\nabla\Psi|^2}{2m_M}+(\omega_L(\mathbf{r})-\omega)|\Psi|^2+F_{\mathrm{so}}(|\Psi|^2)\right), \qquad (10)$$

where m_M is the magnon mass; and $\omega_L(\mathbf{r})=\gamma H(\mathbf{r})$ is the local Larmor frequency, which plays the role of the external potential $U(\mathbf{r})$ in atomic condensates.

The last term F_{so} is analogous to the fourth-order term in the atomic BEC describing the interaction between the atoms of the BEC. In the magnetic subsystem of superfluid ^{3}He, the interaction term is provided by the spin–orbit interaction. Although the structure of superfluid phases of ^{3}He is rather complicated and is described by the multi-component superfluid order parameter [9], the only output needed for investigation of the coherent precession is the structure of the spin–orbit interaction averaged over the precessing state. The resulting spin–orbit interaction F_{so} depends on Ψ and contains the second-order and the fourth-order terms. Thus the spin–orbit interaction provides the effective interaction between magnons. It can be attractive and repulsive, depending on the orientation of spin and orbital degrees with respect to each other and with respect to the magnetic field. By changing the orientation of the orbital part of the order parameter with respect to the magnetic field one is able to regulate the interaction term in experiments and even change its sign. This allows us to observe different states of the coherent precession, i.e. different phases of the magnon BEC.

For a magnon BEC with repulsive interaction, such as ^{3}He-B, all consequences of the phenomenon of superfluidity, which arise due to stiffness of the wavefunction, have been verified experimentally. These include the spin supercurrent which transports the magnetization—the analog of the superfluid mass current in atomic superfluids; the Josephson effect; phonons as collective Goldstone modes; etc.

4. Conclusion

The coherent precession of magnetization in superfluid ^{3}He-B was discovered 20 years ago. Now six different states of magnon BEC have been found in superfluid phases of ^{3}He under different experimental conditions. A magnon BEC may also occur in spin subsystems of other materials with or without magnetic order. It was recently demonstrated in yttrium–iron garnet [10]; it may occur in $CsNiCl_3$ [11] and in cold atomic gases in laser traps; etc. A review of magnon BEC and spin supercurrent phenomena can be found in [12–15].

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